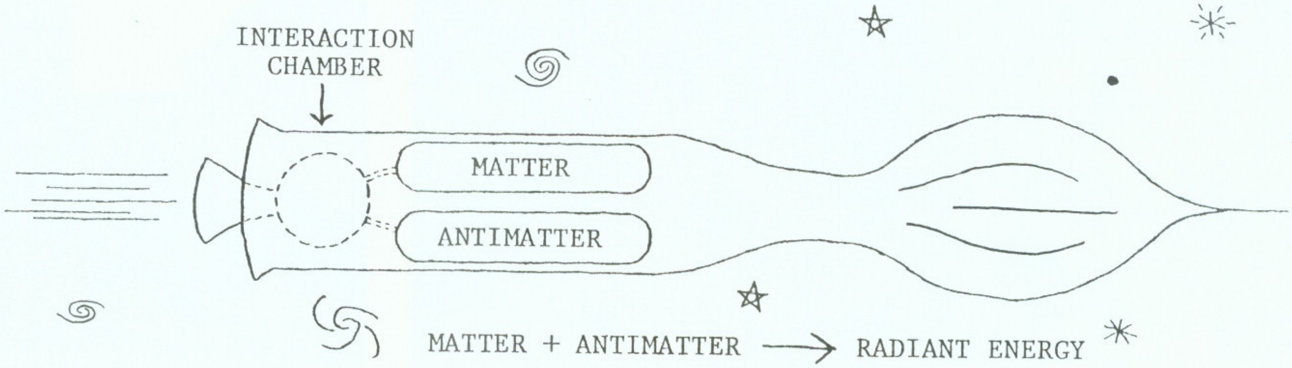
A Rocket Propelled by Radiant Energy

by Scott Ready



Imagine yourself cruising through space in a futuristic rocket powered by light. Aft of you, matter and antimatter are fusing into radiant energy. As this radiant energy jets through your rocket's rear nozzle, it causes an increase in your rocket's forward momentum. As your velocity increases, you wonder how well your rocket can cover intragalactic distances.

You build a clock. On a rod perpendicular to your velocity you fasten two perfect mirrors facing each other. A light beam is made to oscillate between the mirrors, and its time between reflections is used to calibrate any other clocks.

Now suppose a being on a mountain has a mirror clock just like yours and watches you pass over and free fall to Saturn. If during your transit, the light beam in his mirror clock reflects n times, then how many times does the light beam in your clock reflect?

The mountain being sees your clock's light beam trace a zig zag pattern through space (figure 1). Let $\Delta t'$ be the amount of time between two ticks of your mirror clock as seen by you. Then your mirrors are $c\Delta t'$ meters apart, where c is the velocity of light in meters per unit of time.

The value of c is contained in the solution to the Helmholtz electromagnetic wave equation derived from Maxwell's equations. ($c = (\mu_0\epsilon_0)^{-1/2}$ where in outer space $\mathbf{B} = \mu_0\mathbf{H}$, and $\mathbf{D} = \epsilon_0\mathbf{E}$.) Furthermore, Maxwell's equations are the same for all observers. This makes the velocity of light (an electromagnetic wave) the same from all perspectives.

Thus for our mountain being, the length of the hypotenuse of the right triangle in figure 1 is $c\Delta t$ where Δt is the amount of time he measures for one zig or zag to be drawn by the rocket's mirror clock. The horizontal side of the triangle has length $v\Delta t$ where v is the velocity of the rocket relative to the mountain.

By simply applying the pythagorean theorem to this triangle and solving for $\Delta t'$, one gets

$$\Delta t' = \Delta t(1-\beta^2)^{1/2} \quad \text{where} \quad \beta = v/c$$

Notice that as β increases to one, $\Delta t'$ shrinks to zero. Thus if one's spacecraft can achieve any speed less than c, then one may travel to any part of space in any positive amount of time. A 100 year flight from earth to our galactic center (33,000 light years away) would require your average velocity relative to Earth to be 0.999995c.

Consider the spacetime separation between two events which, relative to the rocket, happen at the same place and are separated by $\Delta t'$ units of time. Say the rocket has maintained a constant velocity v relative to the mountain and a velocity v″ relative to a nearby flying saucer. Then the time separation between the two events is $\Delta t = \Delta t'(1-\beta^2)^{-1/2}$ relative to the mountain and $\Delta t'' = \Delta t'(1-\beta''^2)^{-1/2}$ relative to the saucer ($\beta'' = v''/c$). The spacial separation between the events is $\Delta x = v\Delta t$ and $\Delta x'' = v''\Delta t''$ relative to the mountain and saucer respectively. A quick calculation shows that

$$c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 = c^2\Delta t''^2 - \Delta x''^2$$

Thus for these two events, there is a number $\Delta\tau \equiv (c^2\Delta t^2 - \Delta x^2)^{1/2}$ called their *proper time* separation, which like a distance measurement, is not dependent on the particular coordinates or reference frame in which it is calculated. What has been shown for these two events is true for any pair of causally connected events. The rocket and mirror clocks were simply tools for our imagination.

If we had considered two events separated by a positive distance $\Delta x'$ relative to the rocket, then instead of writing $\Delta t = (1-\beta^2)^{-1/2}\Delta t'$ and $\Delta x = v(1-\beta^2)^{-1/2}\Delta t'$ we could have said $\Delta t = (1-\beta^2)^{-1/2}\Delta t' + A\Delta x'$ and $\Delta x = v(1-\beta^2)^{-1/2}\Delta t' + C\Delta x'$ where A and C are functions to be found. The requirement $c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2$ forces $A = \frac{v}{c^2}(1-\beta^2)^{-1/2}$ and $C = (1-\beta^2)^{-1/2}$. Hence

$$\Delta t = (1-\beta^2)^{-1/2}\,\Delta t' + \frac{v}{c^2}(1-\beta^2)^{-1/2}\Delta x'$$

$$\Delta x = v(1-\beta^2)^{-1/2}\,\Delta t' + (1-\beta^2)^{-1/2}\Delta x'$$

These last two equations are the Lorentz transformation equations of Special Relativity.

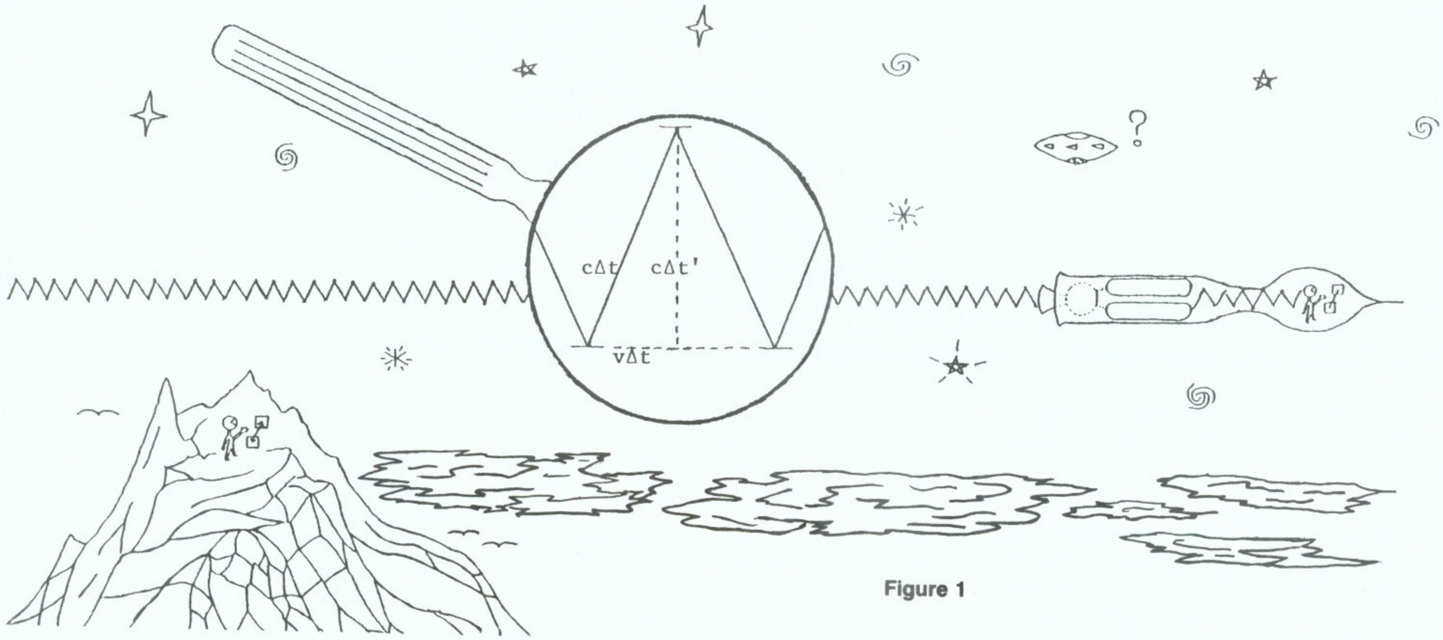


Figure 1

Imagine a flying saucer and the rocket cruising together at a velocity v_1 relative to Earth. A flash of light from the rocket's engines increases its speed relative to the saucer from zero to v_2. Then the rocket's new velocity v relative to Earth does not equal $v_1 + v_2$ but rather (from the Lorentz transformations)

$$v = \frac{\Delta x}{\Delta t} = \frac{v_1(1-\beta_1^2)^{-1/2}\Delta t' + (1-\beta_1^2)^{-1/2}\Delta x'}{(1-\beta_1^2)^{-1/2}\Delta t' + \frac{v_1}{c^2}(1-\beta_1^2)^{-1/2}\Delta x'}$$

$$= \frac{v_1 + \frac{\Delta x'}{\Delta t'}}{1 + \frac{v_1}{c^2}\frac{\Delta x'}{\Delta t'}} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \quad \text{or}$$

$$☆ \qquad \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2} \quad \text{where}$$

$$\beta = v/c,\ \beta_1 = v_1/c, \text{ and } \beta_2 = v_2/c$$

Observe that if v_1 and v_2 are less than c, then so is v. No trick of engineering can cause the rocket to outrun a light signal

In what lies ahead a formula will be derived which computes the change in the rocket's velocity if say 1 kg of matter and 1 kg of antimatter are fused into light and emitted through the rocket's rear nozzle. We need a velocity parameter θ that will compute the final speed of the rocket as a sum (integral) of many velocity parameter increments (differentials) caused by say an intermittent consumption of the rocket's fuel.

Let v be the rocket's velocity relative to Earth. Now define the rocket's velocity parameter θ relative to Earth by

$$\tanh\theta \equiv \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}} = \beta = \frac{v}{c}$$

Then the substitutions $\beta_1 = \tanh\theta_1$ and $\beta_2 = \tanh\theta_2$ into the right-hand side of the starred equation gives one exactly the formula for $\tanh(\theta_1 + \theta_2)$. Thus the starred equation says $\tanh\theta = \tanh(\theta_1 + \theta_2)$. This implies that $\theta = \theta_1 + \theta_2$ since tanh is a one-to-one function.

Thus if one flash of the rocket's engines increases its velocity parameter relative to the saucer from zero to θ, then fifty such flashes will increase its velocity parameter (relative to the saucer) to $50 \cdot \theta$.

Some computing shows that $\sinh\theta \equiv \frac{e^{\theta} - e^{-\theta}}{2} = \beta(1 - \beta^2)^{-1/2}$ and $\cosh\theta \equiv \frac{e^{\theta} + e^{-\theta}}{2} = (1 - \beta^2)^{-1/2}$. Therefore the Lorentz transformation equations can be written as

$$\Delta t = \Delta t' \cosh\theta + \frac{\Delta x'}{c}\sinh\theta$$

$$\Delta x = c\Delta t' \sinh\theta + \Delta x' \cosh\theta$$

The rocket's performance can be calculated by requiring the momentum and energy of the rocket system to be conserved. But what are the correct expressions for the momentum and energy of a high speed system? Experimentally, it is known that the Newtonian expressions for momentum and energy are not conserved at high speed.

Imagine the elastic collision of two identical fragments of matter A and B viewed as in figure 2. B moves perpendicular to the rocket's velocity and sufficiently slow that the Newtonian expression $m\Delta y/\Delta t$ for his momentum relative to the rocket is accepted. The displacements of A and B within the rectangle centered about their collision can be assembled into the triangle of figure 3. The motion of A relative to the saucer is drawn to be the same as the motion of B relative to the rocket. Threfore the *proper times* $2\Delta\tau$ that A and B are within the rectangle are the same. Since B is slow, $c\Delta t$ is a good approximation of $\Delta\tau$. Thus we can write the momentum of B relative to the rocket as $mc\Delta y/\Delta\tau$.

It is natural to consider the momentum of A as tangent to its trajectory. In the elastic collision, the magnitude of A's momentum does not change. Therefore the momentum triangle of figure 4 is similar to the displacement triangle of figure 3. Conservation of momentum implies that the change in the momentum of A (vertical side of the momentum triangle) is exactly opposite to the change in the momentum of B, i.e. $\Delta P_A = -\Delta P_B$. But

$$\Delta P_B = 2P_B = 2m\Delta y/\Delta t = 2mc\Delta y/\Delta\tau$$

Thus by multiplying the sides of the displacement triangle by $mc/\Delta\tau$, one generates the momentum triangle. Hence

$$P_A = mc\Delta r/\Delta\tau = mc\Delta r\,(c^2\Delta t^2 - \Delta r^2)^{-1/2}$$

$$= \frac{mc\dfrac{\Delta r}{c\Delta t}}{\left[1 - \left(\dfrac{\Delta r}{c\Delta t}\right)^2\right]^{1/2}} = mc\beta(1-\beta^2)^{-1/2}$$

$$= mc\sinh\theta$$

where θ is the velocity parameter of A relative to the rocket. As β increases to one, θ and P_A tend to infinity.

Imagine, from the rocket's perspective, the path of an isolated system of mass m through spacetime. Figure 5 shows a displacement vector for the system resolved into a spacial vector and a vector projecting into time. By multiplying the coordinate length of each of these by $mc/\Delta\tau$, one generates a new vector tangent to the system's path called the system's *4-momentum*. The spacial part of this vector is the system's momentum relative to the rocket. What does the part projecting into time represent?

For all observers experiencing weightlessness (no accelerations), the spacial part of the system's *4-momentum* can not change (conservation of momentum). This means that the remaining part $mc\cdot\Delta t/\Delta\tau$ is also conserved. For if not, then any change in $mc\cdot\Delta t/\Delta\tau$ detected by a weightless observer would show up as a change in the momentum of the system when seen from another perspective in uniform motion relative to the first perspective (for further detail, see *Spacetime Physics* pages 111-115). Thus for weightless observers, the quantities $mc\cdot\Delta t/\Delta\tau$ and $mc^3\Delta t/\Delta\tau$ do not change.

Now the quantity $mc^3\Delta t/\Delta\tau$ has the units of energy. Furthermore, it is conserved as energy should be. We therefore call $mc^3\Delta t/\Delta\tau$ the energy E of the system relative to the rocket.

$$E = mc^3\Delta t/\Delta\tau = mc^2\cosh\theta$$

where θ is the velocity parameter of the system relative to the rocket. As θ tends to infinity, so does E. As θ tends to zero, E shrinks to mc^2.

Now imagine a rocket of mass M_0 and a flying saucer free falling together towards our galactic center. Then relative to the saucer, the energy of the rocket is M_0c^2 and its momentum is zero. The rocket blazes its engines and jets, at velocity parameter θ_{ex} relative to the rocket (or saucer), matter of mass m through its rear nozzle. With a new mass M, the rocket edges ahead of the saucer with velocity parameter $\Delta\theta$. Relative to the saucer, the energy of the rocket system is now $mc^2\cosh\theta_{ex} + Mc^2\cosh\Delta\theta$ and its momentum is $-mc\sinh\theta_{ex} + Mc\sinh\Delta\theta$.

The energy and momentum (*4-momentum*) of the rocket system can not change. Therefore

$$M_0c^2 = mc^2\cosh\theta_{ex} + Mc^2\cosh\Delta\theta$$

$$0 = -mc\sinh\theta_{ex} + Mc\sinh\Delta\theta$$

By going from the finite increments $\Delta\theta$ and $M_0 - M$ to the differentials $d\theta$ and dM (and noting that $\cosh d\theta = 1$ and $\sinh d\theta = d\theta$), the conservation of *4-momentum* equations can be rewritten as

$$m\cosh\theta_{ex} = -dM$$

$$m\sinh\theta_{ex} = Md\theta$$

A division of the momentum equation by the energy equation yields

$$\tanh\theta_{ex} = \beta_{ex} = Md\theta/-dM \quad \text{or}$$

$$d\theta = -\beta_{ex}dM/M$$

Upon reducing its mass from M_0 to say M_1, the final velocity parameter θ_F of the rocket relative to the saucer will be the sum (integral) of the incremental (differential) velocity parameter changes caused by each reduction ΔM (dM) of the rocket's mass. That is .

$$\theta_F = \int_0^{\theta_F} d\theta = \int_{M_0}^{M_1} \frac{-\beta_{ex}dM}{M} = \beta_{ex}\ln\frac{M_0}{M_1}$$

The greatest value for θ_F is attained by spacecraft powered by radiant energy. In this case, $\beta_{ex} = 1$ and

$$\theta_F = \ln\frac{M_0}{M_1}$$

Imagine piloting an advanced spacecraft propelled by the emission of radiant energy generated from the interaction of matter and antimatter. After 100 years at the controls, you complete a trip from Earth to our galactic center. Then your velocity parameter relative to Earth must have exceeded 6.5 ($\tanh 6.5 = .999995$). Thus the mass of the fuel consumed on your intragalactic trip must exceed 665 times the combined mass of your rocket, remaining fuel, and self when at the galactic center.

A 100 year flight to our galactic center cannot be entirely glamorous. At rest relative to the stars there is a diffuse medium of one hydrogen atom per cm^3. When our rocket's velocity parameter θ relative to Earth is 6.5, the energy relative to the rocket of each of these hydrogen atoms will be

$$(\text{mass of hydrogen})c^2\cosh 6.5$$

which is 3.11×10^{11} electron volts. Furthermore, at $\theta = 6.5$, a density of one H/cm^3 relative to Earth would be seen as $665H/cm^3$ relative to the rocket. This follows from the second Lorentz transformation equation.

At rest relative to Earth, picture a cube with an edge in line with the rocket's velocity and of length Δx. Relative to the rocket, the separation between the ends of this edge when seen at the same time ($\Delta t' = 0$) will be

$$\Delta x' = \Delta x/\cosh 6.5 = \Delta x/665$$

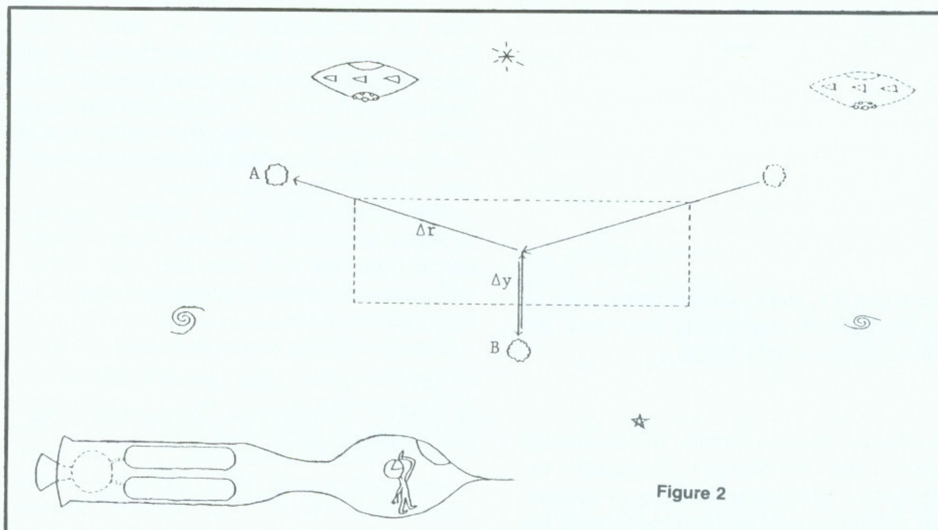
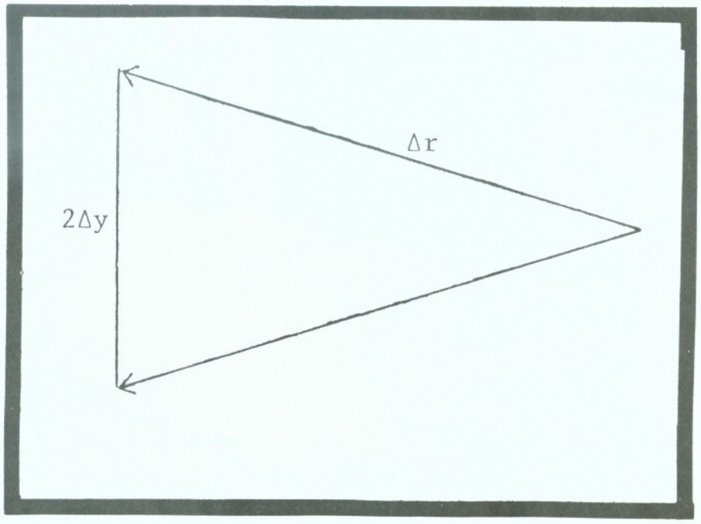


Figure 2



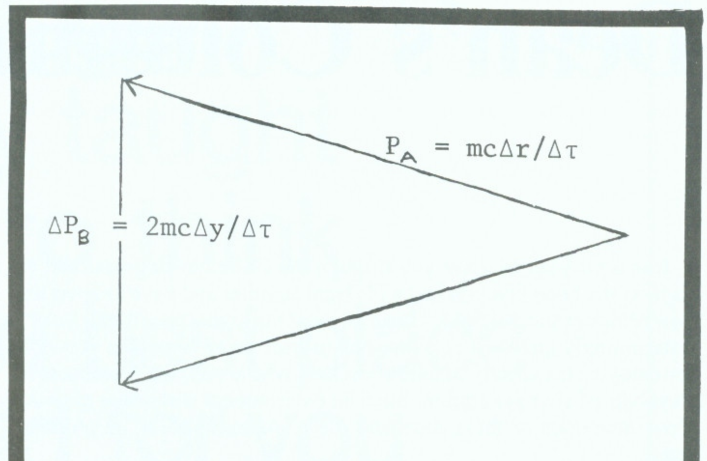


Figure 4 Similar Momentum Triangle

Thus a cube at rest relative to the rocket appears as a long square tube relative to Earth.

The velocity of the hydrogen atoms bombarding the rocket is practically c. Therefore from the perspective of the rocket, each square meter of its forward surface area encounters in one second

$(3\times10^8\ m^3)\ (665 H/cm^3)\ \times$

$(10^6 cm^3/m^3)\ =\ 2\times10^{17}$

hydrogen atoms, each with an energy of 3.11×10^{11} electron volts. This is 30 times more energetic and 200,000 times more concentrated then the beam of a high intensity proton accelerator.

If operators of atomic accelerators on Earth are shielded by a ferroconcrete barrier 3 or 4 meters thick, then how can beings on a 100 year flight to our galactic center be protected?

Draw your own conclusions on the range of intragalactic travel.

SOURCE

Spacetime Physics by John A. Wheeler and Edwin Taylor (1964).

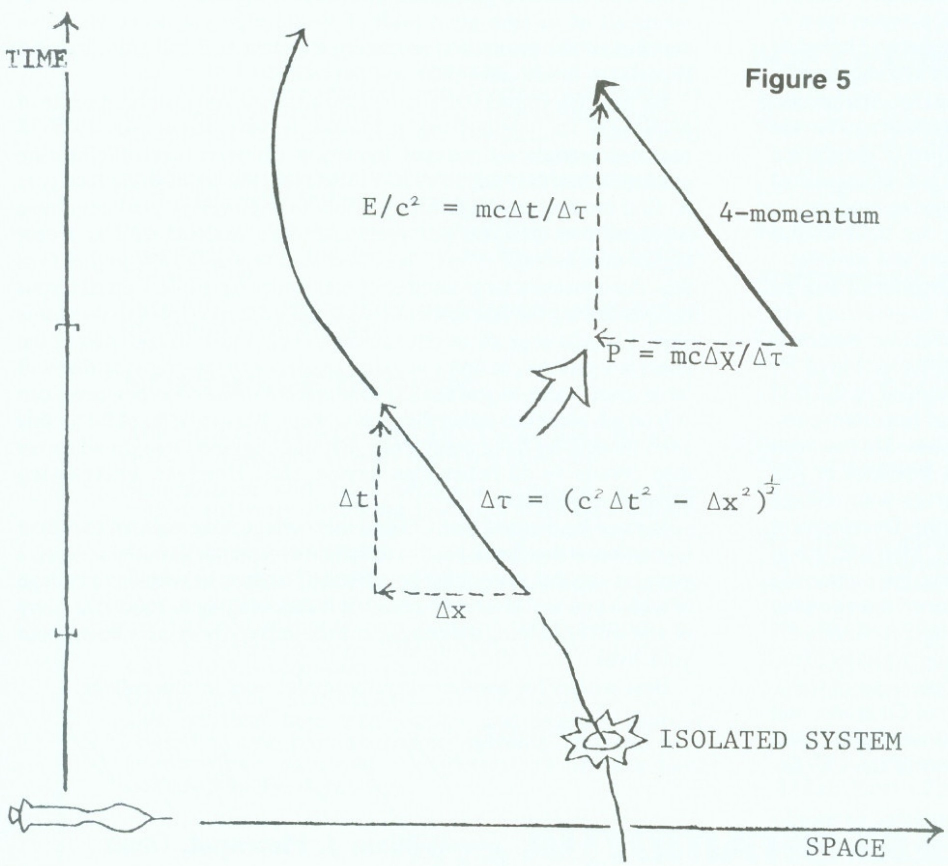


Figure 5

Scott Ready has a masters degree in mathematics and was last seen talking with aliens in an open field.

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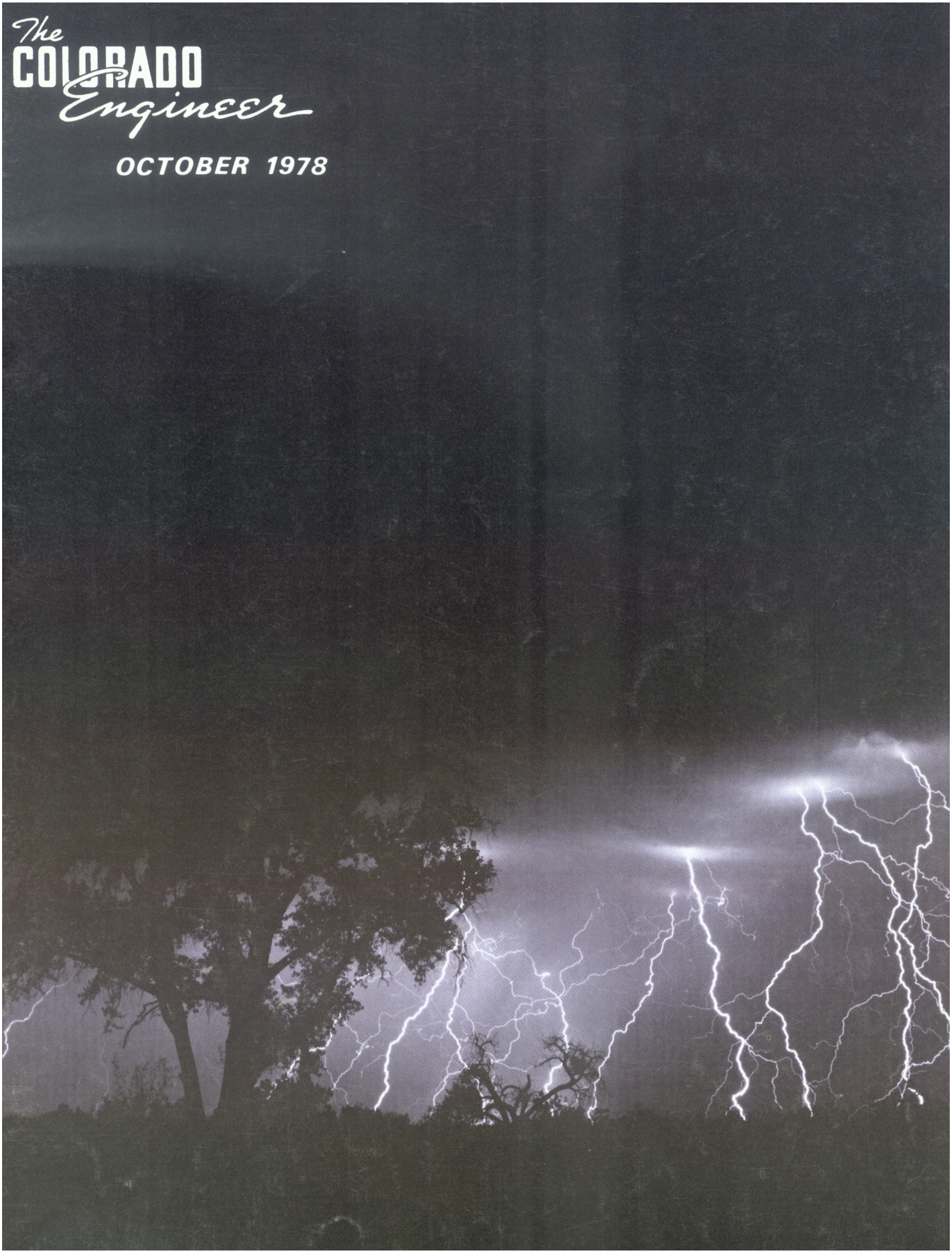
The COLORADO Engineer
OCTOBER 1978

COLORADO ENGINEER

college of engineering / university of colorado / member ecma / volume 75, number 1 /october 1978

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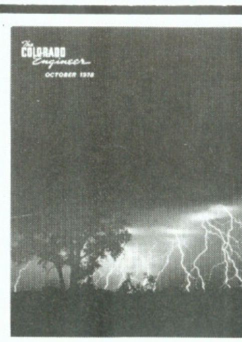

COVER

This striking cover photo of an electrical storm was taken near Fort Collins, Colorado, by professional photographer Marc B. Moore in September of 1976. Entitled "Revenge", the picture is a one and a half minute exposure of a series of lightning strikes. The photographer lives in Fort Collins and exhibits his works throughout the area. For an interesting look at the man-made variety of lightning, turn to our feature article entitled "The Lightning Machine". This unique story introduces our new recurring feature *Focus,* where we zero in on some of the more unusual aspects of research here in the College of Engineering.

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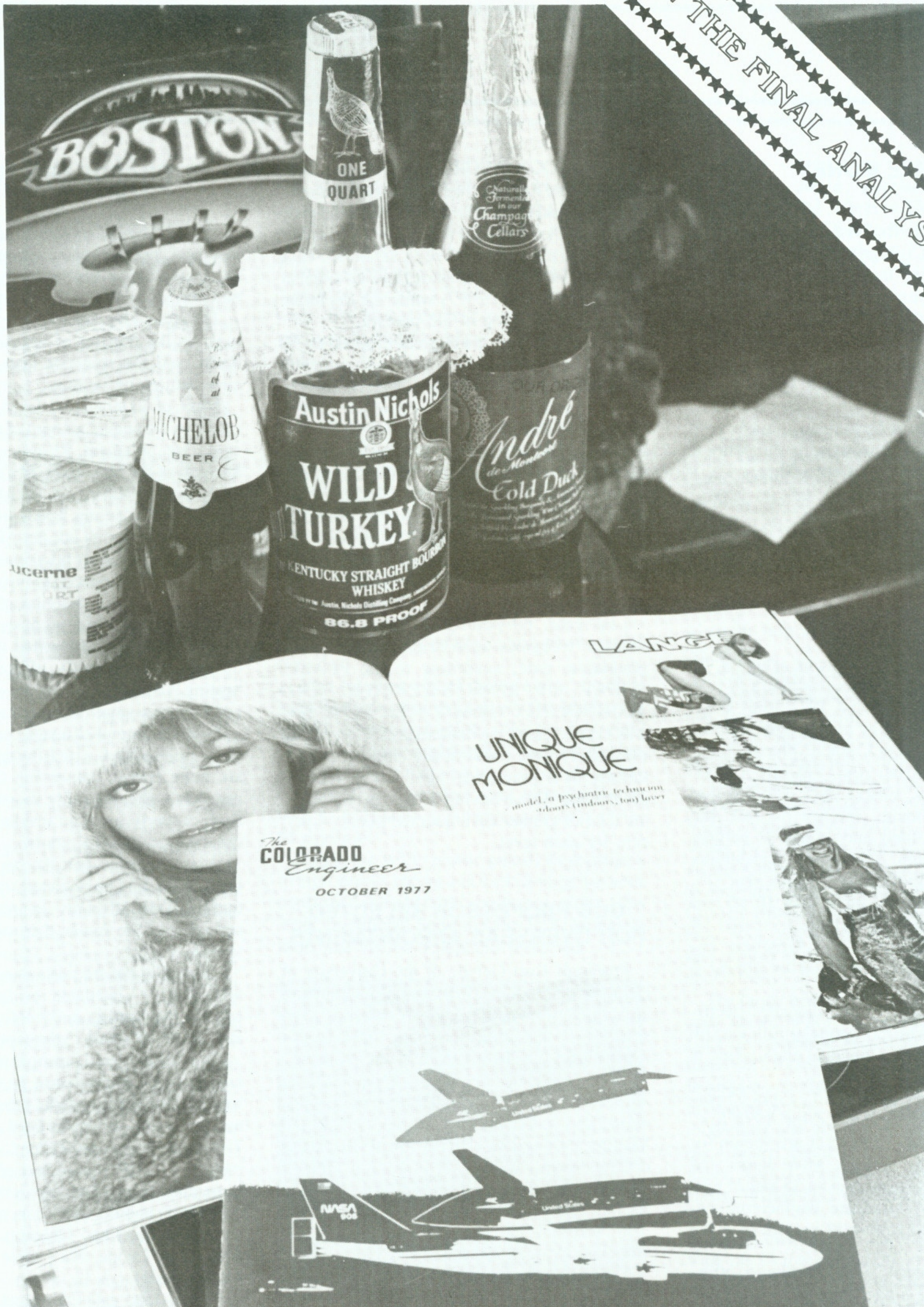
IN THE FINAL ANALYSIS



Richard Luxford

The Necessities of Life